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Translated by J. J. D.

UDC 532.72

DIFFUSION ON A PARTICLE IN A HOMOGENEOUS TRANSLATIONAL-SHEAR FLOW

PMM Vol. 39, № 3, 1975, pp. 497-504

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(Received February 25, 1975)

We consider the problem of a stationary convective diffusion of a substance, dissolved in an incompressible fluid flow on the surface of a particle moving with constant speed in a shear flow field. We assume that the flow over the particle is inertia-free and that there is total absorption of the dissolved component on its surface. In the diffusing boundary layer approximation we determine the concentration field and obtain expressions for the total diffusing stream of a substance on the surface of a solid spherical particle and on the surface of a spherical drop (bubble).

1. The flow field. In a rectangular Cartesian coordinate system fixed to the center of a moving spherical particle (drop) the velocity field of an unperturbed (at large distances from the particle) translational-shear flow can be written in the form

$$\mathbf{v} = \{v_x, v_y, v_z\} = \{-\alpha x, -\alpha y, U + 2\alpha z\} \tag{1.1}$$

Here U is the speed of the unperturbed translational motion of the fluid, α is the shear motion intensity, which may assume both positive and negative values.

We assume that the conditions are satisfied for inertia-free (Stokes) flow over the particle, i.e. we assume that

$$R = Ua / \nu \ll 1, \qquad R_\alpha = |\alpha| a^2 / \nu \ll 1 \tag{1.2}$$

(ν is the coefficient of kinematic viscosity of the flow, and a is the radius of the particle). In the Stokes approximation the distribution of velocities in the fluid in the flow of the stream (1.1) over the particle is a superposition of the velocity fields corresponding to homogeneous translational and homogeneous shear flows over the particle.

We employ the following expressions to describe the distribution of velocities (see, for example, [1, 2])

$$\psi(r, \theta) = \psi_U(r, \theta) + \psi_\alpha(r, \theta) \tag{1.3}$$

$$\psi_U(r, \theta) = \frac{aU}{4}(r - a)\left(2\frac{r}{a} - M_2 - M_2\frac{a}{r}\right)\sin^2\theta$$

$$\psi_\alpha(r, \theta) = \alpha a^3\left(\frac{r^3}{a^3} - \frac{5}{2}M_1 + \frac{3}{2}M_2\frac{a^2}{r^2}\right)\sin^2\theta\cos\theta$$

$$M_1 = \frac{\beta + 2/5}{\beta + 1}, \qquad M_2 = \frac{\beta}{\beta + 1}$$

$$\left(v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \qquad v_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}\right)$$

In writing the expressions (1.3) we used a spherical coordinate system, fixed at the particle center (drop center), in which the angle θ is reckoned from the direction of the translational flow velocity, and β is the ratio of the dynamic viscosities of the drop and of the surrounding fluid (the case of the solid particle corresponds to $\beta \to \infty$). In the case of the drop we assume, in addition, that the Weber number is sufficiently large (the drop remains spherical) and that there are no surface-active substances in the system.

As an example of a flow of the type (1.1) we cite the instance of the flow field over a particle moving in a confuser (diffuser) under conically converging (diverging) stream conditions. In actuality, if we neglect the influence of the walls, the flow far from a particle, moving with speed U_p along the axis of the conical confuser (diffuser), may be described by a superposition of a translational flow with speed $-U_p$ and the flow field (source field) located at the cone vertex. In the coordinate system fixed to the particle center the potential of this flow is given by

$$\varphi_\infty = -U_p r\cos\theta + \frac{v_0 r_O^2}{r_{OM}}$$

$$r_{OM} = (r^2 + r_O^2 - 2rr_O\cos\theta)^{1/2}, \qquad v_0 = \frac{q_0}{2\pi r_O^2(1 - \cos\gamma)}$$

Here r_O is the distance from the particle center to the cone vertex, 2γ is the opening angle of the walls, and q_0 is the fluid outflow rate. Expanding the quantity $1/r_{OM}$ in a series in powers of the ratio r / r_O and limiting ourselves to second order terms, we obtain, apart from a nonessential constant,

$$\varphi_\infty = (v_0 - U_p)\, r\cos\theta + \frac{v_0}{r_O} r^2 \frac{3\cos^2\theta - 1}{2}$$

This potential corresponds to the flow field (1.1) if we put $v_0 - U_p = U$ and $v_0 / r_O = \alpha$.

2. Formulation of the diffusion problem. We assume that the fluid contains a dissolved substance which is completely absorbed on the surface of the particle.

The Schmidt number is large for the majority of liquids, so that, along with the conditions (1.2) for the Reynolds numbers, the following conditions turn out to be satisfied for the Péclet numbers (D is the diffusion coefficient):

$$P = Ua / D \gg 1, \quad P_\alpha = |\alpha| a / D \gg 1$$

Consequently, in determining the diffusion inflow of a substance on the particle surface we can use the diffusion boundary layer approximation and write the convective diffusion equation in the form

$$v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r}\frac{\partial c}{\partial \theta} = D \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial c}{\partial r}\right) \tag{2.1}$$

Here c is the concentration and the velocity components v_r, v_θ are determined from the relations (1.3).

Making a change of variables from r, θ to the variables ψ, θ, we obtain, instead of Eq. (2.1), the equation

$$\frac{\partial c}{\partial \theta} = -D \sin\theta \frac{\partial}{\partial \psi}\left(r^2 \frac{\partial \psi}{\partial r}\frac{\partial c}{\partial \psi}\right) \tag{2.2}$$

In Eq. (2.2) there appears the product $r^2 \partial\psi / \partial r$, which it is necessary to represent in the form of a function of ψ, θ. In the diffusion boundary layer approximation this product is considered in the region $r - a \ll a$. In accord with the relations (1.3) we can represent the principal terms in the series expansions of the stream functions ψ_s and ψ_f in powers of $r - a$ for the solid particle ($\beta \to \infty$) and for the drop ($\beta \ll P^{1/2}$), respectively, in the form

$$\psi_s = \frac{3}{4} U (r-a)^2 (1 + \omega_s \cos\theta)\sin^2\theta, \quad \omega_s = 10\frac{\alpha a}{U} \tag{2.3}$$

$$\psi_f = \frac{1}{2}\frac{aU}{\beta+1}(r-a)(1+\omega_f\cos\theta)\sin^2\theta, \quad \omega_f = 6\frac{\alpha a}{U} \tag{2.4}$$

We wish to emphasize that when $\beta \to 0$ the expression (2.4), and those which follow from it, do not yield a limiting passage to the particle case.

For the determination of the diffusing stream, Eq. (2.2) must be supplemented by boundary conditions, which, for the case considered, have the form

$$\psi \to \pm\infty, \quad c \to c_0; \quad \psi = 0,\ c = 0; \quad \theta = \theta_0,\ c = c_0 \tag{2.5}$$

Here c_0 is the concentration of the dissolved substance away from the particle and θ_0 is the angle defining the position of points on the particle surface at which the diffusion boundary layer originates.

The first of the conditions (2.5) expresses the constancy of the concentration outside of the diffusion boundary layer, the second corresponds to the assumption of complete absorption of the dissolved substance on the particle surface. The third condition is connected with the use of the diffusion boundary layer approximation and expresses the equality of the concentration to the value c_0 on stream lines arriving from infinity and terminating at points where the boundary layer originates.

From the form of the stream function (2.3) for the flow over the particle it follows that the form of the boundary layer and the angle θ_0 depend on the magnitude and the sign of the parameter ω_s characterizing the relative intensity of the shear and translational motions. Four different types of flow over the particle are possible; these are indicated schematically in Fig. 1, where the cases a to d correspond to the following

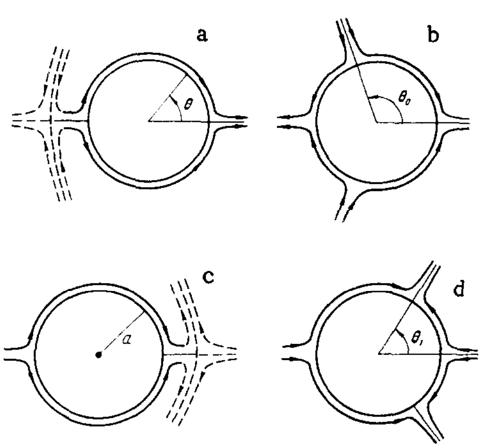

Fig. 1

values of the parameter ω_s and the angle θ_0:

a) $0 \leqslant \omega_s \leqslant 1, \quad \theta_0 = \pi$

b) $\omega_s \geqslant 1, \ \theta_0 = \arccos\left(-\frac{1}{\omega_s}\right)$

c) $-1 \leqslant \omega_s \leqslant 0, \quad \theta_0 = \pi$

d) $\omega_s \leqslant -1, \ \theta_0 = \begin{cases} \theta, \ 0 \leqslant \theta \leqslant \theta_1 \\ \pi, \ \theta_1 < \theta \leqslant \pi \end{cases},$

$$\theta_1 = \arccos\left(-\frac{1}{\omega_s}\right)$$

Similar cases hold for the flow over a drop (in the corresponding expressions for which the quantity ω_f appears instead of the quantity ω_s).

3. Diffusion on a solid particle. In the general case the problem (2.2), (2.3), (2.5) can be represented in the form

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial |\psi_s|}\left(|\psi_s|^{1/2} \frac{\partial c}{\partial |\psi_s|}\right) \tag{3.1}$$

$$t = t(\theta, \omega_s) = \operatorname{sign} \psi_s D a^2 (3U)^{1/2} \int_\theta^{\theta_0} \sin^2 x \, |1 + \omega_s \cos x|^{1/2} dx$$

$$|\psi_s| \to \infty, c \to c_0; \quad \psi_s = 0, c = 0; \quad t = 0, c = c_0$$

The solution of the problem (3.1) has the form

$$c = c_0 \left(\frac{4}{9}\right)^{1/3} \Gamma^{-1}\left(\frac{4}{3}\right) \int_0^z \exp\left(-\frac{4}{9}\eta^3\right) d\eta, \quad z = \frac{|\psi_s|^{1/2}}{t^{1/3}} \tag{3.2}$$

The expression (3.2) enables us to determine differential and integral fluxes of a substance onto the particle surface ; thus, we have

$$j(\theta) = D\left(\frac{\partial c}{\partial r}\right)_{r=a} = D\left(\frac{\partial c'}{\partial |\psi_s|} \frac{\partial |\psi_s|}{\partial r}\right)\Bigg|_{\psi_s=0} \tag{3.3}$$

$$I = 2\pi a^2 \oint j(\theta) \sin\theta \, d\theta$$

We consider individually the four cases indicated above.

a) $0 \leqslant \omega_s \leqslant 1, \ \psi > 0, \ \theta_0 = \pi$. For the local diffusion afflux on the particle surface we obtain

$$j(\theta) = j^\circ (\omega_s + 1)^{-1/6} \frac{\sin\theta \, (1 + \omega_s \cos\theta)^{1/4}}{[A(\pi, p) - A(\theta, p)]^{1/3}} \tag{3.4}$$

$$j^\circ = \frac{c_0}{6^{1/3}\Gamma(4/3)} \left(\frac{UD^2}{a^2}\right)^{1/3}, \ p^2 = \frac{2\omega_s}{\omega_s + 1}$$

$$A(\varphi, k) = \int_0^\varphi \sin^2 x \left(1 - k^2 \sin^2 \frac{x}{2}\right)^{1/2} dx = \frac{8}{15}\left(1 - 3\cos\varphi - \frac{2}{k^2}\right) \times$$

$$\sin\varphi\left(1-k^2\sin^2\frac{\varphi}{2}\right)^{1/2}-\frac{8}{15}\,\frac{(1-k^2)(2-k^2)}{k^4}\,F\left(\frac{\varphi}{2},k\right)+$$
$$\frac{16}{15}\,\frac{k^4-k^2+1}{k^4}\,E\left(\frac{\varphi}{2},k\right)$$

Here $F(\varphi/2, k)$ and $E(\varphi/2, k)$ are elliptic integrals of the first and second kinds. For the total flux we obtain from (3.3) with the aid of (3.4) the expression

$$I(\omega_s)=I^\circ(\omega_s+1)^{1/3}\,[A(\pi,p)]^{2/3},\qquad I^\circ=3\pi a^2 j^\circ \tag{3.5}$$

b) $\omega_s \geqslant 1$, $\theta_0=\arccos(-1/\omega_s)$. In this case the diffusion flux distribution on the surface of the sphere has different forms on the front $(\theta_0\leqslant\theta<\pi)$ and rear $(0<\theta\leqslant\theta_0)$ portions of the sphere. Using the relation

$$A(\varphi,k)=\frac{1}{k^3}\,A\left(2\arcsin\left(k\sin\frac{\varphi}{2}\right),\frac{1}{k}\right)$$

we obtain

$$j(\theta)=j^\circ\frac{(2\omega_s)^{1/2}}{(\omega_s-1)^{2/3}}\,\frac{\sin\theta\,|1+\omega_s\cos\theta|^{1/2}}{\left[A(\pi,q)-A\left(2\arcsin\left(q^{-1}\cos\frac{\theta}{2}\right),q\right)\right]^{1/3}} \tag{3.6}$$

$$q^2=\frac{\omega_s-1}{2\omega_s},\qquad \theta_0\leqslant\theta<\pi$$

$$j(\theta)=j^\circ\frac{(2\omega_s)^{1/2}}{(\omega_s+1)^{2/3}}\,\frac{\sin\theta\,(1+\omega_s\cos\theta)^{1/2}}{\left[A(\pi,p^{-1})-A\left(2\arcsin\left(p\sin\frac{\theta}{2}\right),p^{-1}\right)\right]^{1/3}}$$

$$p^2=\frac{2\omega_s}{\omega_s+1},\qquad 0<\theta\leqslant\theta_0 \tag{3.7}$$

The total diffusion flux of the substance on the particle surface is equal to

$$I(\omega_s)=I^\circ\left\{\frac{(\omega_s-1)^{4/3}}{2\omega_s}\,[A(\pi,q)]^{2/3}+\frac{(\omega_s+1)^{4/3}}{2\omega_s}\,[A(\pi,p^{-1})]^{2/3}\right\} \tag{3.8}$$

Here and henceforth, the values of j°, I° and the function $A(\varphi,k)$ are determined in accord with the relations (3.4) and (3.5).

c) $-1\leqslant\omega_s\leqslant 0$, $\theta_0=\pi$. For the differential and integral fluxes we obtain

$$j(\theta)=j^\circ(|\omega_s|+1)^{-1/6}\,\frac{\sin\theta\,(1-|\omega_s|\cos\theta)^{1/2}}{[A(\pi-\theta,p)]^{1/3}},\qquad p^2=\frac{2|\omega_s|}{|\omega_s|+1} \tag{3.9}$$

$$I(\omega_s)=I^\circ(|\omega_s|+1)^{1/3}\,[A(\pi,p)]^{2/3} \tag{3.10}$$

d) $\omega_s<-1$, $\theta_0=\pi$ for $\theta_1<\theta\leqslant\pi$ and $\theta_0=0$ for $0\leqslant\theta<\theta_1$, $\theta_1=\arccos(1/|\omega_s|)$. For the diffusion flux distribution on the front $(\theta_1<\theta\leqslant\pi)$ and rear $(0\leqslant\theta<\theta_1)$ portions of the sphere we obtain, respectively,

$$j(\theta)=j^\circ\frac{(2|\omega_s|)^{1/2}}{(|\omega_s|+1)^{2/3}}\,\frac{\sin\theta\,(1-|\omega_s|\cos\theta)^{1/2}}{\left[A\left(2\arcsin\left(p\cos\frac{\theta}{2}\right),p^{-1}\right)\right]^{1/3}} \tag{3.11}$$

$$\theta_1<\theta\leqslant\pi$$

$$j(\theta)=j^\circ\frac{(2|\omega_s|)^{1/2}}{(|\omega_s|-1)^{2/3}}\,\frac{\sin\theta\,(|\omega_s|\cos\theta-1)^{1/2}}{\left[A\left(2\arcsin\left(q^{-1}\sin\frac{\theta}{2}\right),q\right]^{1/3}} \tag{3.12}$$

$$q^2 = \frac{|\omega_s| - 1}{2|\omega_s|}, \quad 0 \leqslant \theta < \theta_1$$

The total flux on the whole particle is equal to

$$I(\omega_s) = I^\circ \left\{ \frac{(|\omega_s| - 1)^{4/3}}{2|\omega_s|} [A(\pi, q)]^{2/3} + \frac{(|\omega_s| + 1)^{4/3}}{2|\omega_s|} [A(\pi, p^{-1})]^{2/3} \right\} \quad (3.13)$$

Of greatest interest in the applications is the value of the total diffusion afflux on the particle surface. Using the relations (3.4), (3.5), (3.8), (3.10) and (3.13), we obtain an expression for the Sherwood number, defined along a particle radius, in the form

$$\mathrm{Sh} = \frac{1}{50^{1/3}\Gamma(4/3)} (|\omega_s| + 1)^{1/3} B(p) P^{1/3}, \quad |\omega_s| \leqslant 1 \quad (3.14)$$

$$\mathrm{Sh} = \frac{1}{50^{1/3}\Gamma(4/3)} \left\{ \frac{(|\omega_s| - 1)^{4/3}}{2|\omega_s|} B(q) + \frac{(|\omega_s| + 1)^{4/3}}{2|\omega_s|} B\left(\frac{1}{p}\right) \right\} P^{1/3}, \quad (3.15)$$

$$|\omega_s| \geqslant 1$$

$$B(k) = \left[\frac{(1 - k^2)(2 - k^2)}{k^4} \mathbf{K}(k) - 2\frac{k^4 - k^2 + 1}{k^4} \mathbf{E}(k) \right]^{2/3}$$

Here $\mathbf{K}(k)$ and $\mathbf{E}(k)$ are the complete elliptic integrals of the first and second kind, respectively. The dependence of the quantity $\mathrm{Sh}\, P^{-1/3}$ on $|\omega_s|$ is shown in Fig. 2.

The results obtained here confirm the essential influence of the shear flow on the mass transfer. In the special case $|\omega_s| = 0$ the relations (3.4), (3.5) or (3.9), (3.10) yield the results obtained in [1] for translational flow over the particle; when $|\omega_s| \to \infty$ the relations (3.6) – (3.8) and (3.11) – (3.13) correspond to the results deduced in [3, 4] for a particle in a homogeneous shear flow.

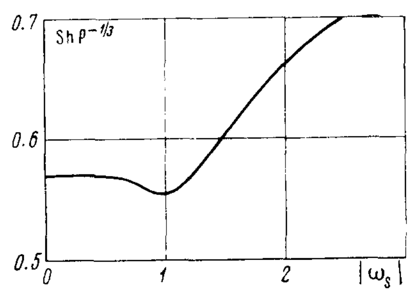

Fig. 2

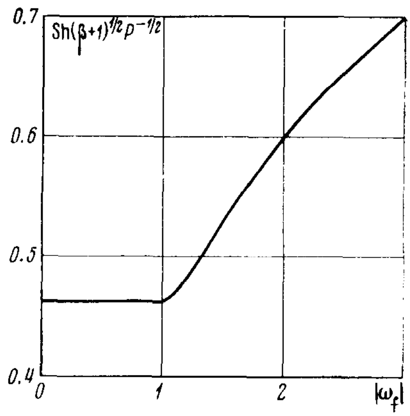

Fig. 3

4. Diffusion on a drop (bubble). We can write the problem (2.2), (2.4), (2.5) in the form

$$\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial \psi_f^2}, \quad t = \frac{Da^3 U}{2(\beta + 1)} \int_\theta^{\theta_0} (1 + \omega_f \cos\theta) \sin^3\theta \, d\theta \quad (4.1)$$

$$|\psi_f| \to \infty, \ c \to c_0; \quad \psi_f = 0, \ c = 0; \quad t = 0, \ c = c_0$$

Its solution is

$$c(z) = \frac{2c_0}{\sqrt{\pi}} \int_z^0 \exp(-\eta^2)\, d\eta, \qquad z = \frac{|\psi_f|}{2t^{1/2}} \tag{4.2}$$

Using the expression (4.2), we determine differential and integral diffusion fluxes on the drop surface in accord with the expressions (3.3) for the various cases of flow over the drop. For the cases (a) and (c) the diffusion flux distribution over the drop surface has the form

$$j(\theta) = j^\circ \frac{(1-\cos\theta)\ (1+\omega_f\cos\theta)}{[{}^1/_3 (2-\cos\theta) - {}^1/_4\omega_f (1-\cos\theta)^2]^{1/2}}, \qquad j^\circ = c_0 \left[\frac{DU}{2\pi a\ (\beta+1)}\right]^{1/2} \tag{4.3}$$

Integrating the expression (4.3) over the drop surface, we obtain

$$I = 4\sqrt{\frac{2\pi}{3}}\, c_0 \left[\frac{Da^3U}{\beta+1}\right]^{1/2} \tag{4.4}$$

It is evident that the presence of the shear flow has no effect on the total diffusion afflux of the substance on the drop surface for $|\omega_f| \leqslant 1$.

In case (b) the diffusion flux distribution on the front and rear portions of the drop surface is described by the expression

$$j(\theta) = j^\circ \frac{\omega_f^{2/3} \sin^2\theta}{[{}^1/_4\omega_f^2\ (2-\cos^2\theta) - {}^1/_{12}\,(1-2\omega_f\cos\theta)]^{1/2}} \tag{4.5}$$

Integrating the expression (4.5) over the whole drop surface, we obtain

$$I = 4\pi a^2 j^\circ\, [\zeta_+^{1/2}(\omega_f) + \zeta_-^{1/2}(\omega_f)] \tag{4.6}$$

$$\zeta_\pm(\omega_f) = \pm\frac{2}{3} + \frac{\omega_f}{4} + \frac{1}{2\omega_f} - \frac{1}{12\omega_f^3}$$

In case (d) the diffusion flux distributions on the front and rear portions of the drop have, respectively, the forms

$$j(\theta) = j^\circ \frac{(1-\cos\theta)\ (1-|\omega_f|\cos\theta)}{[{}^1/_3(2-\cos\theta) + {}^1/_4|\omega_f|\,(1-\cos\theta)^2]^{1/2}} \tag{4.7}$$

$$\theta_1 \leqslant \theta < \pi, \qquad \theta_1 = \arccos\,(1/|\omega_f|)$$

$$j(\theta) = j^\circ \frac{(1+\cos\theta)\ (|\omega_f|\cos\theta - 1)}{[-{}^1/_3(2+\cos\theta) + {}^1/_4|\omega_f|\,(1+\cos\theta)^2]^{1/2}}, \qquad 0 < \theta \leqslant \theta_1 \tag{4.8}$$

Integrating the expressions (4.7) and (4.8) over the front and rear portions of the drop, we obtain for the integral flux the result

$$I = 4\pi a^2 j^\circ\, [\zeta_+^{1/2}(|\omega_f|) + \zeta_-^{1/2}(|\omega_f|)] \tag{4.9}$$

Of greatest interest in practical applications is the value of the integral flux. Using the relations (4.4), (4.6) and (4.9), we obtain for the Sherwood number the results

$$\mathrm{Sh} = \sqrt{\frac{2}{3\pi}}\,(\beta+1)^{-1/2}P^{1/2}, \qquad |\omega_f| \leqslant 1 \tag{4.10}$$

$$\mathrm{Sh} = \sqrt{\frac{1}{8\pi}}\left[\left(1+\frac{1}{|\omega_f|}\right)^{3/2}\left(|\omega_f| - \frac{1}{3}\right)^{1/2} + \left(1-\frac{1}{|\omega_f|}\right)^{3/2}\left(|\omega_f| + \frac{1}{3}\right)^{1/2}\right](\beta+1)^{-1/2}P^{1/2}, \qquad |\omega_f| \geqslant 1 \tag{4.11}$$

The dependence of the quantity $\mathrm{Sh}\,(\beta+1)^{1/2}P^{-1/2}$ on $|\omega_f|$ is shown in Fig. 3. This quantity remains constants for $|\omega_f| \leqslant 1$ and increases as $|\omega_f|$ increases for $|\omega_f| > 1$.

In conclusion we note that in the limiting cases of homogeneous translational ($\omega_f \to 0$) and homogeneous shear ($|\omega_f| \to \infty$) flows the expressions (4.10) and (4.11) agree with those obtained earlier in [1, 3, 4].

The authors thank G. Iu. Stepanov for valuable comments.

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Translated by J. F. H.

UDC 539.3

STATIC AND DYNAMIC CONTACT PROBLEMS WITH COHESION

PMM Vol. 39, № 3, 1975, pp. 505-512

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(Received March 20, 1974)

Systems of integral equations, originating in plane and axisymmetric contact problems of elasticity theory in the case of cohesion of a stamp to a body, are studied. A method is developed which is based on factorization of matrix functions of a special kind and its foundation is given. Applications of the method in static and dynamic problems are presented. The method is especially effective in dynamic contact problems of stamp vibration on the surface of a layered medium or a cylinder.

Other methods of solving contact problems with cohesion have been proposed in [1 – 8].

1. Systems of integral equations of the following two kinds

$$\sum_{n=1}^{2} r_{mn} q_n = f_m(x), \qquad x \in \Omega,\ m = 1, 2 \tag{1.1}$$

$$r_{mn} q_n = \frac{1}{2\pi} \int_{-a}^{a} \int_{\sigma} R_{mn}(u)\, e^{iu(x-\xi)} du\, q_n(\xi)\, d\xi, \quad \Omega \equiv [-a, a] \tag{1.2}$$